Plausible Reasoning from Conditionals with System P and Rational Monotony

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Extended Abstract[4]

For the modelling of human reasoning, much effort has been put into studying how humans reason with conditionals of the form *If A, then usually B*, establishing a connection between the antecedent A and the consequent B that is plausible, but that still allows for exceptions. While humans are typically very good at dealing with such conditionals, there is no generally accepted formal framework characterizing this reasoning. As a base for human reasoning, in this paper we explore the formal space of reasoning from a set of conditionals, called a belief base, that is obtained by employing two main axiomatic requirements, system P [1, 7] and *rational monotony* (RM) [7, 10].

In logic-based knowledge representation, inductive reasoning from (conditional) belief bases has been a major task for a long time. Especially in the field of non-monotonic reasoning, research often deals with inferences that can be drawn from a set of given defeasible rules, and the resulting inference relations are assessed in terms of broadly accepted axiomatic properties, like system P [7]. Beyond the inference relations, the seminal papers [9, 11] put the role of the belief base into the focus of reasoning methods, proposing closure operations for reasoning from defeasible rule bases that have inspired many other works on non-monotonic reasoning since then. In particular, *Rational Closure, (RC)* [9] (or equivalently *system Z* [11]) are inductive inference operators that can be characterized by a certain closure of a belief base under *rational monotony* (RM) [7, 10] and exhibit desirable properties. Every inference relation satisfying system P and (RM) is induced by a ranked model (or equivalently a total preorder (TPO) on worlds) [8]. An inference relation satisfying system P is called *preferential* and is induced by a *preferential model* [7]. Both system P and (RM) have benefits and drawbacks:

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- System P is generally seen as a kind of gold standard which a non-monotonic inference relation should fulfil. However, inference only with the axioms of system P (p-entailment) is very skeptical because it takes *all* preferential models of a belief base Δ into account. Therefore, system P on its own is often perceived to be too weak for drawing informative inferences.
- If $A \mid\!\sim C$, the postulate (RM) licences the entailment $A \wedge B \mid\!\sim C$ for *every* B as long as from A we cannot defeasibly entail the negation of B. Therefore, because no other condition on B is required, (RM) is often perceived to be too strong.

Thus, one would expect inference operators to comply with system P while possibly licensing additional conditional entailments. The postulate (Classic Preservation) [4] requires that the inductive inference operator licenses an entailment of the form $A \mid\!\sim_\Delta \bot$ only if $A \mid\!\sim^p_\Delta \bot$, i.e., if it is a p-entailment. Note that $A \mid\!\sim_\Delta \bot$ causes all models of A to be completely infeasible. Thus, (Classic Preservation) ensures that an inference operator considers a world infeasible if an only if system P does so.

The postulate (RC Extension) [4] restricts the closure under (RM) to the belief base Δ and thus makes a difference between beliefs explicitly given in Δ and implicit beliefs derived from Δ by non-monotonic entailment. This distinction between explicit and implicit beliefs perfectly fits the basic idea of inductive inference operators [6], which map a belief base Δ to a complete inference relation induced by Δ. Inference relations satisfying (RM), *Classic Preservation* (CP), and (RC Extension) can be semantically characterized by ranked models that are *rank preserving* with respect to the Z-ranking [4].

In our paper [5], we explore the field of inference relations involving system P respectively (RM) as limiting characterizations, and extend the work started in [4] by dropping the rather strong requirement of (RM). Instead, we consider more general classes of so-called RCP inductive inference operators, i.e., inductive inference operators satisfying (RC Extension) and (Classic Preservation). For RCP inductive inference operators that satisfy system P (RCP preferential inductive inference operators) we show that these are characterized by *Z-rank refining* preferential models, where *Z-rank refining* is a newly introduced adaption of *rank preserving* to preferential models. The intuition of Z-rank refining is that the preferential model respects and possibly refines the structure on worlds that is induced by Z-ranking functions κ^z_Δ.

While preferential models are more general than TPOs, they are also more complex. Between the class of preferential inference relations and the class of inference relations induced by TPOs there is the class of inductive inference operators induced by strict partial orders (SPOs) on worlds. SPOs on worlds are more expressive than TPOs but less complex than preferential models: e.g., for signatures of size $|\Sigma| = 2$ there are 75 TPOs, 219 SPOs, and 485 (non-equivalent) preferential models on the four Σ-worlds [2, 3]. Thus, to fill the gap between TPOs and preferential models we also consider the class of RCP inductive inference operators induced by SPOs on worlds, called RCP *SPO-representable* inductive inference operators. We show that RCP SPO-representable inductive inference operators are characterized by *Z-rank refining* SPOs on worlds. Furthermore, we investigate inference relations induced by SPOs on formulas and show that such inductive inference operators satisfy RCP if they are based on Z-rank refining SPOs on formulas. Thus, our work extends [4] in different directions, in particular by providing characterization theorems for different classes of RCP inductive inference operators.

To summarize, we investigated RCP inductive inference oparators, i.e., inductive inference operators satisfying (RC Extension) and (Classic Preservation). Doing this we focused on SPO-representable inference relations, i.e., inference relations that can be obtained from SPOs on worlds. We proved a characterization theorem stating that RCP preferential inductive inference operators can be characterized by Z-rank refining preferential models. Additionally, we introduced the class of SPO-representable inductive inference operators, which prove to be central within a map of inductive inference operators and showed a characterization theorem expressing that RCP SPO-representable inductive inference operators can be characterized by Z-rank refining SPOs on worlds.

Future work includes to further investigate instances of rational inductive inference operators; especially the characterization of such inference operators by their properties will be of interest.

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