On the instantiation of argument-incomplete argumentation frameworks

Antonio Yuste-Ginel¹, Carlo Proietti²

¹Universidad Complutense de Madrid, Departamento de Lógica y Filosofía Teórica, Spain ²Consiglio Nazionale delle Ricerche, Istituto di Linguistica Computazionale «A. Zampolli» (CNR-ILC), Italy

Abstract

Argument-incomplete argumentation frameworks provide an intuitive way of representing uncertainty in argumentative contexts. It is however possible that, taking structured argumentation as a reference point, the general assumptions of these models present the same risks of hasty generalization attributed to some abstract argumentation models, as they do not have a structured counterpart. Here, we focus on a specific instantiation of argument-incomplete argumentation frameworks: rooting the uncertainty about arguments in the uncertainty about the application of ASPIC+-inference rules. We show (Proposition 1) that the abovementioned risk is concrete. Therefore a more fine-grained representation of uncertainty at the abstract level is needed, which we provide with implicative argument-incomplete argumentation frameworks and prove to work (Theorem 1).

Keywords

Abstract argumentation, Structured argumentation, Uncertainty, Incompleteness

1. Introduction and motivation

Encoding uncertainty about arguments and attacks is key for applying formal argumentation in several contexts, including strategic ones such as modelling opponents in a debate [1]. Recent literature on abstract argumentation witnesses models of different inspiration [2, 3, 4, 5, 6]. Yet, one question is whether such abstract models are adequate to capture proper argumentative uncertainty. This echoes more general concerns about abstract argumentation models, insofar as they open to assumptions and generalisations that are unjustified or meaningless at the structured level (as shown in [7, 8, 9]).

To address this question in more precise terms, we consider argument-incomplete abstract argumentation frameworks (arg-IAAFs) [10, 11, 12] as our abstract model of qualitative uncertainty. Further, we take ASPIC⁺ [7], with its notion of *structured argumentation frameworks* (SAFs), as our underlying formalism for structured argumentation. Following the suggestions by [12], uncertainty can be generated either by uncertain inference rules or by incomplete preference profiles. Here we focus on the first option: the uncertainty of an argument is explained by the uncertainty of whether one or more inference rules of this argument must be applied.

More procedurally, we define rule-incomplete structured argumentation frameworks (rul-ISAFs) as incomplete extensions of SAFs, and as natural counterparts of arg-IAAFs in abstract

ACLAI23 (also submitted to AI^3)



© 0 2022 Copyright for this paper by its authors. Use permitted under Creative Commons License Attribution 4.0 International (CC BY 4.0). CEUR Workshop Proceedings (CEUR-WS.org)

argumentation. As a first negative result (Proposition 1), we show that there are rul-ISAFs that cannot be represented abstractly as IAFs. However, correspondence is retrieved (Theorem 1) if we instead consider a refinement of arg-IAAFs, that we name *implicative argument-incomplete abstract argumentation framework* (imp-arg-IAAF).

Section 2 provides the necessary formal background on abstract and structured argumentation, as well as the newly defined notions. In Section 3 we prove our main results. Section 4 sketches our current research directions.

2. Background

A (Dung) **abstract argumentation framework** (AAF) is a directed graph $\langle \text{Arg}, \text{Def} \rangle$ where Arg is a set of *arguments* and Def \subseteq Arg × Arg is a *defeat relation* among them.

An **argument-incomplete abstract argumentation framework** (arg-IAAF) is a tuple $\langle \operatorname{Arg}^F, \operatorname{Arg}^2, \operatorname{Def} \rangle$ where Arg^F and Arg^2 are two pairwise disjoint sets of arguments and $\operatorname{Def} \subseteq (\operatorname{Arg}^F \cup \operatorname{Arg}^2) \times (\operatorname{Arg}^F \cup \operatorname{Arg}^2)_2$.

A **completion** of $\langle \operatorname{Arg}^F, \operatorname{Arg}^2, \operatorname{Def} \rangle$ is any AAF $\langle \operatorname{Arg}^*, \operatorname{Def}^* \rangle$ s.t.:

- $\operatorname{Arg}^{F} \subseteq \operatorname{Arg}^{*} \subseteq \operatorname{Arg}^{F} \cup \operatorname{Arg}^{?}$.
- $\operatorname{Def}^* = \operatorname{Def}_{\upharpoonright \operatorname{Arg}^*}$.

As announced arg-IAAFs will fail to be expressive enough for capturing uncertain inference rules. That's why we need a more refined formalism. An **implicative argumentincomplete abstract argumentation framework** (imp-arg-IAAF) is $\langle \operatorname{Arg}^F, \operatorname{Arg}^2, \operatorname{Def}, \Delta \rangle$ where $\langle \operatorname{Arg}^F, \operatorname{Arg}^2, \operatorname{Def} \rangle$ is an argument-incomplete abstract argumentation framework and $\Delta \subseteq \operatorname{Arg}^2 \times \operatorname{Arg}^2$ is a set of implicative dependencies. Informally, $(A, B) \in \Delta$ means that *B* appears in a completion whenever *A* does. Formally, a **completion** of $\langle \operatorname{Arg}^F, \operatorname{Arg}^2, \operatorname{Def}, \Delta \rangle$ is any AAF $\langle \operatorname{Arg}^*, \operatorname{Def}^* \rangle$ s.t.:

- $\langle \operatorname{Arg}^*, \operatorname{Def}^* \rangle$ is a completion of $\langle \operatorname{Arg}^F, \operatorname{Arg}^?, \operatorname{Def} \rangle$.
- For all $(X, Y) \in \Delta$ if $X \in Arg^*$, then $Y \in Arg^*$.

Note that imp-arg-IAAFs can be seen as a restricted class of constrained incomplete AFs [13, 14] or as a restricted class IAFs with correlations [15].

Finally, we introduce the ASPIC⁺ notions [16] that are relevant to our study. A **structured argumentation framework** (SAF) is a tuple $AF = \langle L, \overline{\cdot}, R, n, K, Arg, Att, \preceq \rangle$ where each component is defined as follows:

- L is a formal language.
- $\overline{\cdot}$: L $\rightarrow 2^{L}$ is a contrary function. We say that:
 - φ is contrary of ψ iff $\psi \in \overline{\varphi}$ but $\varphi \notin \overline{\psi}$.
 - φ is contradictory of ψ (noted $\varphi = -\psi$) iff $\psi \in \overline{\varphi}$ and $\varphi \in \overline{\psi}$.
 - each $\varphi \in L$ has at least one contradictory.
- $R = R_s \cup R_d$ with $R_s \cap R_d = \emptyset$ is a set of inference rules (sequences of elements of L). R_s represents strict rules while R_d represents defeasible rules.

- \mathfrak{n} : $R_d \rightarrow L$ is a partial naming function for defeasible rules.
- K ⊆ L is a knowledge base, assumed to be split into two disjoint subsets K_n (axioms) and K_p (ordinary premises).
- Arg is the set of arguments of \$AF, which is defined inductively together with some auxiliary functions: Sub(·) (returns the subarguments of a an argument), Prem(·) (returns the premisses of a an argument), Conc(·) (returns the conclusion of a an argument), and TopRule(·) (returns the last rule employed in the construction of a an argument). We have that A ∈ Arg iff:
 - $A = [\varphi]$ if $\varphi \in K$, with $Prem(A) = Conc(A) = \{\varphi\}$, $Sub(A) = \{\varphi\}$ and TopRule(A) is left undefined.
 - $-A = [A_1, ..., A_n \twoheadrightarrow \varphi] \text{ if } A_i \text{ is an argument for } 1 \le i \le n \text{ and} \\ (\operatorname{Conc}(A_1), ..., \operatorname{Conc}(A_n), \varphi) \in \mathsf{R}_s, \text{ with } \operatorname{Prem}(A) = \bigcup_{1 \le i \le n} \operatorname{Prem}(A_i), \operatorname{Conc}(A) = \varphi, \\ \operatorname{Sub}(A) = \{A\} \cup \bigcup_{1 \le i \le n} \operatorname{Sub}(A_i), \operatorname{TopRule}(A) = (\operatorname{Conc}(A_1), ..., \operatorname{Conc}(A_n), \varphi).$
 - $-A = [A_1, ..., A_n \Rightarrow \varphi] \text{ if } A_i \text{ is an argument for } 1 \le i \le n \text{ and} \\ (\operatorname{Conc}(A_1), ..., \operatorname{Conc}(A_n), \varphi) \in \mathsf{R}_d, \text{ with } \operatorname{Prem}(A) = \bigcup_{1 \le i \le n} \operatorname{Prem}(A_i), \operatorname{Conc}(A) = \varphi, \\ \operatorname{Sub}(A) = \{A\} \cup \bigcup_{1 \le i \le n} \operatorname{Sub}(A_i), \operatorname{TopRule}(A) = (\operatorname{Conc}(A_1), ..., \operatorname{Conc}(A_n), \varphi).$
- Att \subseteq Arg × Arg is the **attack relation** of SAF. ASPIC⁺ allows for three kinds of attacks. We say that *A* **attacks** *B* (i.e., (*A*, *B*) \in Att) iff *A* undermines, rebuts or undercuts *B*, where:
 - − *A* undermines *B* (on *B'*) iff Conc(*A*) ∈ $\overline{\varphi}$ for some *B'* = φ ∈ Prem(*B*) and φ ∈ K_{*p*}.
 - A **rebuts** B (on B') iff Conc(A) $\in \overline{\varphi}$ for some B' =Sub(B) of the form $B'_1, ..., B'_n \Rightarrow \varphi$.
 - A undercuts B (on B') iff Conc $(A) \in \overline{\mathfrak{n}}(\operatorname{TopRule}(B'))$ for some $B' = \operatorname{Sub}(B)$ with TopRule $(B') \in \mathbb{R}_d$.
- $\leq \subseteq$ Arg × Arg is a preference relation among arguments, with $\prec = \leq \setminus \leq^{-1}$ its strict counter-part.

Given $AF = \langle L, \overline{,} R, n, K, Arg, Att, \leq \rangle$, we use L(SAF) to denote L and apply the same convention for the rest of the components. Let $A, B \in Arg(AF)$, we say that A **defeats** B iff: (i) A undercuts B, or (ii) A undermines/rebuts B (on B') and $A \not\prec B'$. The set of all defeats for a given AF is denoted Def(SAF). Finally, given AF, the **Dung's argumentation framework associated to** AF is just DAF(AF) = (Arg(AF), Def(AF)).

3. Structured frameworks with uncertain inference rules

As mentioned, here we consider the set of rules of a given structured argumentation framework as a source of uncertainty for the presence of arguments. In the same spirit of IAFs, one can define a **rule-incomplete structured argumentation framework** as a tuple rul-ISAF = $\langle L, \bar{,} R, n, K, Arg, Att, \leq \rangle$, where every component is just as in a SAF except from the set of rules R, which is split into four pairwise disjoint subsets $R = R_s^F \cup R_s^2 \cup R_d^F \cup R_d^2$, representing respectively certain strict rules, uncertain strict rules, certain defeasible rules and uncertain defeasible rules. Then, a **rule-completion of** rul-ISAF is any SAF^{*} = $\langle L, \bar{,} R^*, n, K, Arg^*, Att^*, \leq^* \rangle$ where:

•
$$\mathbf{R}^{*} = \mathbf{R}^{*}_{s} \cup \mathbf{R}^{*}_{d}$$
 is s.t.:

 $\begin{array}{l} - \ \mathsf{R}_s^F \subseteq \mathsf{R}_s^* \subseteq (\mathsf{R}_s^F \cup \mathsf{R}_s^?); \\ - \ \mathsf{R}_d^F \subseteq \mathsf{R}_d^* \subseteq (\mathsf{R}_d^F \cup \mathsf{R}_d^?). \end{array}$

• Arg* and Att* are the set of arguments and attacks generated by R*.

• $\preceq^* = \preceq_{\upharpoonright Arg^*}$.

We denote by rul-completions(ISAF) the set of rule-completions of rul-ISAF. Finally, let rul-ISAF be given, its set of **completions** is simply defined as:

completions(rul-ISAF) = $\{DAF(SAF^*) | SAF^* \in rul-completions(rul-ISAF)\}$.

Our research question can be now put in more precise terms: is the set of completions of a rul-ISAF always equal to the set of completions of some (arg-)IAAF? The answer is negative. Formally,

Proposition 1. Given rul-ISAF, it is not necessarily the case that there is arg-IAAF s.t.: completions(rul-ISAF) = completions(arg-IAAF).

Proof. A simple counterexample is provided by considering any rul-ISAF where L is the language of propositional logic containing atoms p, q and r, \exists is classical negation, with $R_s = R_s^? = \emptyset$, $R_d = \{(q, r)\}$, and $R_d^? = \{(p, q)\}$, $K_s = \emptyset$, $K_p = \{p\}$, and $\preceq = \emptyset$. Then, completions(rul-ISAF) has two members, namely $\langle \{[p]\}, \emptyset \rangle$ and $\langle \{[p], [[p] \Rightarrow q], [[[p] \Rightarrow q] \Rightarrow r]\}, \emptyset \rangle$. Clearly, for basic cardinality reasons, no arg-IAAF has an isomorphic set of completions, since a completion with one argument and a completion with three arguments would force the presence of another completion with two arguments.

As mentioned, one way to interpret this result is that nothing guarantees that the completion of an (arg-)IAAF is subargument closed, e.g., that $p \Rightarrow q \Rightarrow r$ forces the presence of $p \Rightarrow q$, as in Proposition 1.

Theorem 1. Let rul-ISAF be a rul-ISAF, there exists an imp-arg-IAAF s.t.

completions(rul-ISAF) = completions(imp-arg-IAAF).

Sketch of the proof. Let rul-ISAF = $(L, \bar{\cdot}, R, \mathfrak{n}, K, Arg, Att, \preceq)$ be a rul-ISAF, we will use two of its completions in the proof:

- \mathbb{SAF}^F is the rule-completion whose set of rules is $\mathbb{R}^F = \mathbb{R}^F_s \cup \mathbb{R}^F_d$ (i.e., the set of certain rules).
- \mathbb{SAF}^{max} is the rule-completion whose set of rules is $\mathbb{R}^{max} = \mathbb{R}^F_s \cup \mathbb{R}^F_d \cup \mathbb{R}^?_s \cup \mathbb{R}^?_d$ (i.e., the completion generated by using all certain and uncertain rules).

Now, we are going to build the target imp-arg-IAAF $\langle Arg^F, Arg^?, Def, \Delta \rangle$:

•
$$\operatorname{Arg}^{F} = \operatorname{Arg}(\mathbb{SAF}^{F}).$$

- Arg[?] = Arg(\mathbb{SAF}^{max}) \ Arg(\mathbb{SAF}^{F}).
- $Def = Def(SAF^{max}).$
- $\Delta = \{(X, Y) \in \operatorname{Arg}^2 \times \operatorname{Arg}^2 | Y \in \operatorname{Sub}(X)\}.$

We show that both directions of the equality completions(rul-ISAF) = completions($\langle \operatorname{Arg}^{F}, \operatorname{Arg}^{?}, \operatorname{Def}, \Delta \rangle$) hold:

[⊆] Suppose (Arg^{*}, Def^{*}) \in completions(rul-ISAF), which amounts by definition of completions to

(H1) $(\operatorname{Arg}^*, \operatorname{Def}^*) = \mathbb{DAF}(\mathbb{SAF}^*)$ for some $\mathbb{SAF}^* \in \operatorname{rul-completions}(\mathbb{ISAF})$

Hence, we just need to check that $(\operatorname{Arg}^*, \operatorname{Def}^*)$ satisfies the conditions for being a completion of $\langle \operatorname{Arg}^F, \operatorname{Arg}^?, \operatorname{Def}, \Delta \rangle$, namely: (a) $\operatorname{Arg}^F \subseteq \operatorname{Arg}^* \subseteq \operatorname{Arg}^F \cup \operatorname{Arg}^?$; (b) $\operatorname{Def}^* = \operatorname{Def}_{\uparrow \operatorname{Arg}^*}$; and (c) for all $(X, Y) \in \Delta$ if $X \in \operatorname{Arg}^*$, then $Y \in \operatorname{Arg}^*$.

To do so, we need to establish the following claims, whose proof we omit:

Lemma 1. Let SAF and SAF' only differ in their set of inference rules. Then:

- 1. $R(SAF) \subseteq R(SAF')$ implies $Arg(SAF) \subseteq Arg(SAF')$.
- 2. $R(SAF) \subseteq R(SAF')$ implies $Def(SAF) = Def(SAF')_{\upharpoonright Arg(SAF)}$.
- 3. Arg(\mathbb{SAF}) is closed under subarguments (i.e., $X \in \operatorname{Arg}(\mathbb{SAF})$ implies $\operatorname{Sub}(X) \subseteq \operatorname{Arg}(\mathbb{SAF})$).

Based on how we defined our target imp-arg-IAAF and H1, condition 1 of Lemma 1 entails (a), since $R(\mathbb{SAF}^F) \subseteq R(\mathbb{SAF}^*) \subseteq R(\mathbb{SAF}^{max})$. For the same reason, (b) follows from condition 2 by $R(\mathbb{SAF}^*) \subseteq R(\mathbb{SAF}^{max})$. Finally, condition 3 entails (c) by how we set Δ .

[⊇] Suppose (Arg^{*}, Def^{*}) ∈ completions((Arg^{*F*}, Arg[?], Def, Δ)). Then conditions (a)-(c) above are satisfied. We need to show that (Arg^{*}, Def^{*}) ∈ completions(rul-ISAF), which means, by definition, that for some SAF ∈ rul-completions(rul-ISAF) we have DAF(SAF) = (Arg^{*}, Def^{*}). Now, let us consider SAF' = (L, ¬, R', n, K, Arg', Att', ≤') where R' = {TopRule(X) | X ∈ Arg^{*}}, Arg' = Arg^{*}, Att' = Att_{|Arg'} and ≤'=≤_{|Arg'} (it is easy to check that SAF' is actually a SAF). Using conditions (a)-(c) above, one can show that either SAF' is the rule-completion of ISAF we are looking for, or that there is one rule-completion SAF^{*} with R(SAF') ⊆ R(SAF^{*}) s.t. DAF(SAF^{*}) = DAF(SAF').¹

4. Future work

A natural completion of the present work is to investigate whether the existential claim of Theorem 1 holds in the other direction as well, i.e. for every imp-arg-IAAF there is a rul-ISAF with an isomorphic set of completions. Together with Theorem 1, this would amount to a characterization result for the class of rul-ISAFs. We conjecture that this is in fact the case, but the proof is not as direct as that of Theorem 1 and we leave it for future work.²

Another task is to investigate the second root of uncertainty mentioned by [12, Section 8.1.]: incomplete defeats based on incomplete preference profiles. Here again, we think a negative result, analogous to Proposition 1, obtains. As for the analogous to Theorem 1, we think that a restricted version of attack-IAAFs ([17]) with correlations could work at the abstract level.

¹The second case emerges from the fact that there could be (potentially) useless rules within R, these are, rules that do not take part in the construction of any argument within a rule-completion.

²Among other subtleties, note that for Theorem 1 we played with the fact that the domain of an imp-arg-IAAF $\operatorname{Arg}^{F} \cup \operatorname{Arg}^{?}$ could be any set (even a set of ASPIC⁺ arguments). However, this does not apply for ISAFs (its arguments are always ASPIC⁺ arguments), so for the other direction of the theorem one needs to define a (straightforward) isomorphism from rul-ISAF completions to imp-arg-IAAF completions.

References

- N. Oren, T. J. Norman, Arguing using opponent models, in: P. McBurney, I. Rahwan, S. Parsons, M. N. (Eds.), International Workshop on Argumentation in Multi-Agent Systems, volume 6057 of *LNCS*, Springer, 2009, pp. 160–174.
- H. Li, N. Oren, T. J. Norman, Probabilistic argumentation frameworks, in: Proceedings of TAFA, volume 7312 of *LNCS*, Springer, 2011, pp. 1–16. doi:10.1007/978-3-642-29184-5\ _1.
- [3] Y. Dimopoulos, J. Mailly, P. Moraitis, Control argumentation frameworks, in: S. A. McIlraith, K. Q. Weinberger (Eds.), Proceedings of the Thirty-Second AAAI Conference on Artificial Intelligence, AAAI Press, 2018, pp. 4678–4685. URL: https://www.aaai.org/ocs/index.php/AAAI/AAAI18/paper/view/16639. doi:10.1609/aaai.v32i1.11583.
- [4] D. Baumeister, D. Neugebauer, J. Rothe, H. Schadrack, Verification in incomplete argumentation frameworks, Artificial Intelligence 264 (2018) 1–26. doi:10.1016/j.artint.2018. 08.001.
- [5] T. Rienstra, M. Thimm, N. Oren, Opponent models with uncertainty for strategic argumentation, in: International Joint Conference in Artificial Intelligence, AAAI Press, 2013.
- [6] C. Proietti, A. Yuste-Ginel, Dynamic epistemic logics for abstract argumentation, Synthese 199 (2021) 8641–8700. doi:10.1007/s11229-021-03178-5.
- [7] H. Prakken, M. De Winter, Abstraction in argumentation: Necessary but dangerous, in: Computational Models of Argument, IOS Press, 2018, pp. 85–96.
- [8] A. Cohen, S. Parsons, E. I. Sklar, P. McBurney, A characterization of types of support between structured arguments and their relationship with support in abstract argumentation, International Journal of Approximate Reasoning 94 (2018) 76–104.
- [9] H. Prakken, Relating abstract and structured accounts of argumentation dynamics: the case of expansions, in: Proceedings of the International Conference on Principles of Knowledge Representation and Reasoning, volume 19, 2023, pp. 562–571.
- [10] D. Baumeister, J. Rothe, H. Schadrack, Verification in argument-incomplete argumentation frameworks, in: T. Walsh (Ed.), Algorithmic Decision Theory, Springer International Publishing, Cham, 2015, pp. 359–376.
- [11] D. Baumeister, D. Neugebauer, J. Rothe, H. Schadrack, Complexity of verification in incomplete argumentation frameworks, in: S. A. McIlraith, K. Q. Weinberger (Eds.), Proceedings of the Thirty-Second AAAI Conference on Artificial Intelligence, (AAAI-18), AAAI Press, 2018, pp. 1753–1760. doi:10.1609/aaai.v32i1.11562.
- [12] D. Baumeister, M. Järvisalo, D. Neugebauer, A. Niskanen, J. Rothe, Acceptance in incomplete argumentation frameworks, Artificial Intelligence 295 (2021) 103470. doi:10.1016/j. artint.2021.103470.
- [13] J.-G. Mailly, Constrained incomplete argumentation frameworks, in: J. Vejnarová, N. Wilson (Eds.), Symbolic and Quantitative Approaches to Reasoning with Uncertainty, volume 12897 of *LNCS*, Springer, 2021, pp. 103–116. doi:10.1007/978-3-030-86772-0_8.
- [14] A. Herzig, A. Yuste-Ginel, Abstract argumentation with qualitative uncertainty: An analysis in dynamic logic, in: P. Baroni, C. Benzmüller, Y. N. Wáng (Eds.), Logic and Argumentation, volume 13040 of *LNCS*, Springer, 2021, pp. 190–208. doi:10.1007/

```
978-3-030-89391-0_11.
```

- [15] B. Fazzinga, S. Flesca, F. Furfaro, Reasoning over argument-incomplete AAFs in the presence of correlations, in: Z.-H. Zhou (Ed.), International Joint Conference in Artificial Intelligence, IJCAI Organization, 2021, pp. 189–195. URL: https://doi.org/10.24963/ijcai. 2021/27. doi:10.24963/ijcai.2021/27.
- [16] S. Modgil, H. Prakken, A general account of argumentation with preferences, Artificial Intelligence 195 (2013) 361–397.
- [17] B. Fazzinga, S. Flesca, F. Furfaro, Reasoning over Attack-incomplete AAFs in the Presence of Correlations, in: M. Bienvenu, G. Lakemeyer, E. Erdem (Eds.), Proceedings of the 18th International Conference on Principles of Knowledge Representation and Reasoning, 2021, pp. 301–311. URL: https://doi.org/10.24963/kr.2021/29. doi:10.24963/kr.2021/29.